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k-SMOOTHNESS ON POLYHEDRAL BANACH SPACES

BY

SUBHRAJIT DEY, ARPITA MAL and KALLOL PAUL (Kolkata)

Abstract. We characterize k-smoothness of an element on the unit sphere of a finite-dimensional polyhedral Banach space. Then we study k-smoothness of an operator $T \in \mathbb{L}(\ell_\infty^n, \mathbb{Y})$, where $\mathbb{Y}$ is a two-dimensional Banach space with the additional condition that T attains its norm at each extreme point of $B_{\ell_\infty^n}$. We also characterize k-smoothness of an operator from ℓ_∞^3 to ℓ_1^3.

1. Introduction. The study of k-smoothness plays an important role in identifying the structure of the unit ball of a Banach space. The papers [1, 2, 3, 4] contain the study of k-smooth points of many Banach spaces. Several papers, including [1, 3, 4, 5, 6, 7, 9] study k-smoothness of operators on different spaces. In [7], the present authors have obtained a relation between k-smoothness and extreme points of the unit ball of a polyhedral Banach space.

The purpose of this paper is to characterize the order of smoothness of an element of the unit sphere of a finite-dimensional polyhedral Banach space; we also study k-smoothness of an operator defined between polyhedral Banach spaces. Let us first fix the notation and terminology.

The letters $\mathbb{X}$, $\mathbb{Y}$ denote Banach spaces. Throughout the paper we assume all Banach spaces considered to be real. We denote the unit ball and the unit sphere of $\mathbb{X}$ respectively by $B_{\mathbb{X}}$ and $S_{\mathbb{X}}$, i.e., $B_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| \leq 1\}$ and $S_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| = 1\}$. Let $\mathbb{L}(\mathbb{X}, \mathbb{Y})$ denote the space of all bounded linear operators between $\mathbb{X}$ and $\mathbb{Y}$. For $T \in \mathbb{L}(\mathbb{X}, \mathbb{Y})$, M_T denotes the collection of all unit vectors of $\mathbb{X}$ at which T attains its norm, i.e., $M_T = \{x \in S_{\mathbb{X}} : \|Tx\| = \|T\|\}$. For a set A, the cardinality of A is denoted by $|A|$. The dual space of $\mathbb{X}$ is denoted by $\mathbb{X}^*$.

An element x of a convex set C is said to be an *extreme point* of C if $x = (1-t)y + tz$ for some $y, z \in C$ and $t \in (0,1)$ implies that $y = z = x$. The set of all extreme points of C is denoted by $\mathrm{Ext}(C)$. For $x, y \in \mathbb{X}$, let $L[x, y] = \{tx + (1-t)y : 0 \leq t \leq 1\}$ and $L(x, y) = \{tx + (1-t)y : 0 < t < 1\}$.

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Some Remarks on Orthogonality of Bounded Linear Operators

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We explore the relation between the orthogonality of bounded linear operators in the space of operators and that of elements in the ground space. To be precise, we study if $T, A \in \mathbb{L}(\mathbb{X}, \mathbb{Y})$ satisfy $T \perp_B A$, then whether there exists $x \in \mathbb{X}$ such that $Tx \perp_B Ax$ with $\|x\| = 1$, $\|Tx\| = \|T\|$, where $\mathbb{X}, \mathbb{Y}$ are normed linear spaces. In this context, we introduce the notion of Property P_n for a Banach space and illustrate its connection with orthogonality of a bounded linear operator between Banach spaces. We further study Property P_n for various polyhedral Banach spaces.

Keywords: Orthogonality, linear operators, norm attainment, polyhedral Banach spaces.

2010 Mathematics Subject Classification: Primary 46B20; secondary 47L05.

1. Introduction

The purpose of the present article is to continue the study of orthogonality properties of bounded linear operators between Banach spaces, in light of the seminal result obtained by Bhatia and Šemrl [1] regarding orthogonality of linear operators on Euclidean spaces. Let us first establish the relevant notations and the terminologies in this context.

The letters $\mathbb{X}$ and $\mathbb{Y}$ denote Banach spaces. Throughout the article, we work only with real Banach spaces. Let $B_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| \leq 1\}$ and $S_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| = 1\}$ denote the unit ball and the unit sphere of $\mathbb{X}$ respectively. Let $E_{\mathbb{X}}$ denote the set of all extreme points of $B_{\mathbb{X}}$. For a set $\mathcal{S} \subset \mathbb{X}$, $|\mathcal{S}|$ denotes the cardinality of $\mathcal{S}$. Let $\mathbb{L}(\mathbb{X}, \mathbb{Y})$ denote the Banach space of all bounded linear operators from $\mathbb{X}$ to $\mathbb{Y}$, endowed with the usual operator norm. We write $\mathbb{L}(\mathbb{X}, \mathbb{Y}) = \mathbb{L}(\mathbb{X})$, if $\mathbb{X} = \mathbb{Y}$. For a bounded linear operator $T \in \mathbb{L}(\mathbb{X}, \mathbb{Y})$, let M_T denote the norm attainment set of T, i.e., $M_T = \{x \in S_{\mathbb{X}} : \|Tx\| = \|T\|\}$. The notion of Birkhoff-James orthogonality in a Banach space is well-known and is used extensively in the study of the geometry of Banach spaces. For $x, y \in \mathbb{X}$, x is said to be *orthogonal* to y in the sense of Birkhoff-James [2], written as $x \perp_B y$, if $\|x + \lambda y\| \geq \|x\|$ for all $\lambda \in \mathbb{R}$. Similarly, for

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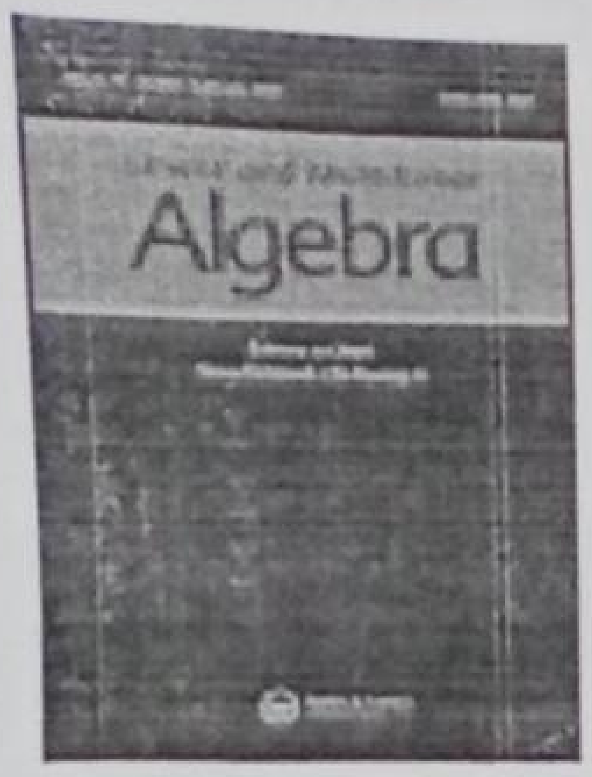

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Characterization of *k*-smoothness of operators defined between infinite-dimensional spaces

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Characterization of extreme contractions through *k*-smoothness of operators

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ABSTRACT

We characterize extreme contractions defined between finite-dimensional polyhedral Banach spaces using *k*-smoothness of operators. As application of results obtained, we explicitly compute the number of extreme contractions in some special Banach spaces. Our approach, in this paper, in studying extreme contractions lead to the improvement and generalization of previously known results.

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Extreme contraction; *k*-smoothness; linear operators; polyhedral Banach space; weak L-P property

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1. Introduction

The study of extreme contractions and smoothness of operators between Banach spaces are two classical and fertile areas of research in Banach space theory. While the characterization of extreme contractions defined between Hilbert spaces is well-known [1–4], the characterization of the same is still elusive, in the general setting of Banach spaces. There are several papers including [5–18] that deal with the study of extreme contractions of operators defined between some special Banach spaces. The purpose of this paper is to study extreme contractions between polyhedral Banach spaces and explore interesting connections between the order of smoothness of an operator and extreme contraction. In particular, we generalize and improve on the results obtained in [14] in an elegant way. Before proceeding further, we first establish the notations and terminologies.

We denote the Banach spaces by the letters $\mathbb{X}$ and $\mathbb{Y}$. Throughout the paper, we assume that the Banach spaces are real. $|A|$ denotes the cardinality of a set A. An element x of a convex set A is said to be an extreme point of A, if $x = ty + (1-t)z$ for some $t \in (0,1)$ and $y, z \in A$ implies that $x = y = z$. The set of all extreme points of a convex set A is denoted by $\mathrm{Ext}(A)$. The unit ball and the unit sphere of $\mathbb{X}$ are denoted by $B_{\mathbb{X}}$ and $S_{\mathbb{X}}$, respectively, that is, $B_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| \leq 1\}$ and $S_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| = 1\}$. $\mathbb{L}(\mathbb{X}, \mathbb{Y})$ denotes the

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Some remarks on orthogonality of bounded linear operators-II

Debmalya Sain, Anubhab Ray, Subhrajit Dey, and Kallol Paul

Abstract. Let $\mathbb{X}, \mathbb{Y}$ be normed linear spaces. We continue exploring the validity of the Bhatia–Šemrl (BŠ) Property: "An operator $T \in \mathbb{L}(\mathbb{X}, \mathbb{Y})$ satisfies Bhatia–Šemrl Property if for any $A \in \mathbb{L}(\mathbb{X}, \mathbb{Y})$, $T \perp_B A$ implies that there exists a unit vector $x \in \mathbb{X}$ such that $\|Tx\| = \|T\|$ and $Tx \perp_B Ax$." A pair of normed linear spaces $(\mathbb{X}, \mathbb{Y})$ is said to be a BŠ pair if for every $T \in \mathbb{L}(\mathbb{X}, \mathbb{Y})$, T satisfies the BŠ Property if and only if $M_T = D \cup (-D)$, where D is a non-empty connected subset of $S_{\mathbb{X}}$. We show that $(\ell_1^n, \mathbb{Y})$ is a BŠ pair for any normed linear space $\mathbb{Y}$, and also obtain some other results in this context. In particular, using the notion of norm attainment set, we characterize the space ℓ_∞^3 among all 3-dimensional polyhedral Banach spaces whose unit ball have exactly eight extreme points.

1. Introduction

Birkhoff–James orthogonality plays a central role in determining the geometry of normed linear spaces in general, and spaces of operators, in particular. One of the most interesting aspects of Birkhoff–James orthogonality is the relation between orthogonality of operators and that of norming elements in the ground space. The purpose of this paper is to continue the investigation of a certain property from [7], as mentioned in the abstract. Before proceeding further, let us fix the notations and the terminologies.

Letters $\mathbb{X}$ and $\mathbb{Y}$ denote normed linear spaces. Throughout the present article, we will assume the underlying scalar field to be $\mathbb{R}$. Let $B_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| \leq 1\}$ and $S_{\mathbb{X}} = \{x \in \mathbb{X} : \|x\| = 1\}$ denote the unit ball and the unit sphere of $\mathbb{X}$, respectively. Let $B[x, r] = \{z \in \mathbb{X} : \|x - z\| \leq r\}$ and $B(x, r) = \{z \in \mathbb{X} : \|x - z\| < r\}$ denote the closed ball and the open ball centered at x and radius $r > 0$, respectively. For a subset A of $\mathbb{X}$, let $|A|$ denote the cardinality of A. Let $\mathbb{L}(\mathbb{X}, \mathbb{Y})$ be the normed space of all bounded

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Geometric properties of operator spaces endowed with the numerical radius norm

Subhrajit Dey[1] · Arpita Mal[2] · Kallol Paul[3]

Received: 17 July 2022 / Accepted: 18 November 2022

Abstract

We characterize operators having equal operator norm and numerical radius norm. Then we explore a generalized notion of smoothness on $\mathbb{L}(\mathbb{X})_w$, the space of bounded linear operators on a real finite-dimensional Banach space $\mathbb{X}$ endowed with the numerical radius norm. Furthermore, we explore extreme contractions in $\mathbb{L}(\mathbb{X})_w$, whenever $\mathbb{X}$ is a finite-dimensional real polyhedral Banach space. Finally, we obtain the structure of the set of extreme points in the dual space of $\mathbb{L}(\mathbb{X})_w$, where $\mathbb{X}$ is a two-dimensional polygonal Banach space.

Keywords Numerical radius norm · Nr-smoothness of order k · Linear operator · Banach space · Nr-extreme contraction

Mathematics Subject Classification Primary 46B20 · Secondary 47L05

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