2023

MATHEMATICS — HONOURS

Paper: SEC-B-1

(Mathematical Logic)

Full Marks: 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

(Notations and symbols have their usual meanings.)

1.	Choose	the	correct	option	and	justify	your	answer	:
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 $(1+1) \times 10$

- (a) The proposition $(\alpha \to \beta) \to (\neg \beta \to \neg \alpha)$ is a
 - (i) tautology

(ii) contradiction

(iii) contingency

- (iv) both (i) and (ii).
- (b) $\sim (p \leftrightarrow q)$ is logically equivalent to
 - (i) $q \leftrightarrow p$

(ii) $p \leftrightarrow \sim q$

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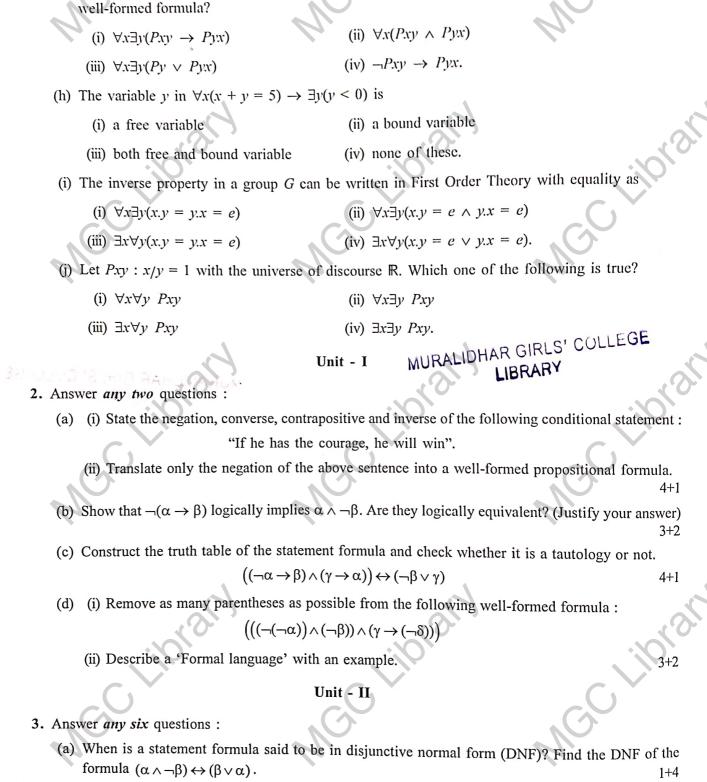
(iii)
$$\sim p \leftrightarrow \sim q$$

- (iv) $\sim q \leftrightarrow \sim p$
- (c) Which one of the following is not a well-formed propositional formula?
 - (i) $(\alpha \to \beta) \to (\neg \beta)$
- (ii) $(\alpha \leq \beta) \rightarrow (\neg \beta \vee \neg \alpha)$
- (iii) $(\alpha \land \beta) \rightarrow (\neg \gamma \rightarrow \neg \delta)$
- (iv) $(\gamma \rightarrow \delta) \wedge (\alpha \leftrightarrow \neg \alpha)$
- (d) Conjunctive normal form of the formula $\alpha \leftrightarrow \beta$ is
 - (i) $(\neg \alpha \lor \beta) \land (\neg \beta \lor \alpha)$
- (ii) $(\alpha \vee \beta) \wedge (\neg \beta \vee \alpha)$
- (iii) $(\neg \alpha \lor \beta) \land (\beta \lor \alpha)$
- (iv) $(\alpha \vee \neg \beta) \wedge (\beta \vee \neg \alpha)$.
- (e) The contrapositive of the well-formed formula $\alpha \to \neg \beta$ (up to logically equivalent) is
 - (i) $\beta \rightarrow \neg \alpha$

(ii) $\neg \beta \rightarrow \neg \alpha$

(iii) $\beta \rightarrow \alpha$

- (iv) $\beta \vee \neg \alpha$.
- (f) If the truth value of the well-formed formula $\alpha \to \beta$ is False then the truth value of $\neg \beta \land \alpha$
 - (i) is False
 - (ii) is True
 - (iii) is sometimes True and sometimes False
 - (iv) cannot be determined.



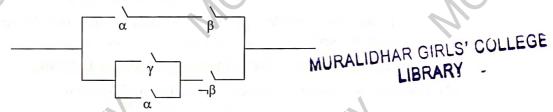
(2)

(g) Let P be a two-place predicate symbol in Predicate Calculus. Which one of the following is not a

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(b) Write the equivalent statement form representing the following circuit. Find the simplest equivalent circuit for it.



- (c) Prove that the only binary connectives that alone are adequate for the construction of all truth functions are \downarrow and \mid .
- (d) Find a derivation for each of the theorems in Propositional Calculus:

2+3

- (i) $\vdash \neg \neg \alpha \rightarrow \alpha$
- (ii) $\vdash (\alpha \rightarrow \beta) \rightarrow (\neg \beta \rightarrow \neg \alpha)$.
- (e) Let \mathcal{F} be the set of all well-formed formulas in Propositional Calculus. A binary relation \equiv is defined on \mathcal{F} as follows:
 - $\alpha \equiv \beta$ if and only if $\vdash \alpha \to \beta$ and $\vdash \beta \to \alpha$, for all $\alpha, \beta \in \mathcal{F}$. Prove that \equiv is an equivalence relation on \mathcal{F} . Also, prove that the quotient set $\mathcal{F}/\equiv \{[\alpha]\equiv : \alpha \in \mathcal{F}\}$ forms a Poset with respect to the relation \leq , where \leq is defined by $[\alpha]\equiv \leq [\beta]\equiv$ if and only if $\vdash \alpha \to \beta$, for all $[\alpha]\equiv$, $[\beta]\equiv \in \mathcal{F}/\equiv$.
- (f) State and prove Deduction Theorem of Propositional Calculus.

5

- (g) Define 'valuation function' in Propositional Calculus. How can this function be extended over the set of all well-formed formulas of Propositional Calculus? Hence, show that the rule Modus Ponens in Propositional Calculus is a valid argument.
 1+2+2
- (h) Define inconsistent and absolute inconsistent of a set $\Gamma \subseteq \mathcal{F}$, the set of all well-formed formulas of Propositional Calculus. Show that Γ is inconsistent if and only if Γ is absolute inconsistent.

1+1+3

- (i) Prove that any consistent set Γ of propositional formulas can be extended to a maximal consistent set.
- (i) Write down the rule of inference of Propositional Calculus.
 - (ii) Examine the validity of the following argument:

"If I study hard, then I will stand first in the examination.

I work hard. Therefore, I will stand first in the examination."

1+4

Unit - III

- 4. Answer any four questions :
 - (a) Define alphabet, term and well-formed formulas in Predicate logic.

1+2+2

(b) (i) Mention each occurrence of variables in the following well-formed formula as free or bound: $\forall x \forall z ((x-y=z) \rightarrow \exists y (y+x < z)).$

Is there any variable in the above formula which is both free and bound variable? (Justify your answer)

(ii) Write a closed well-formed formula in Predicate Calculus.

(3+1)+1

- (c) Translate the following sentences into well-formed formulas:
 - (i) No philosopher understands politics.
 - (ii) Anyone who knows Julia loves her.
 - (iii) Some people respect everyone.

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1+2+2

(d) Over the universe of mammals, let

Px: x is a person

Lxy: x loves y

Translate the following well-formed formulas into ordinary English. Note that ordinary English does not use variables.

- (i) $\forall x (Px \rightarrow \exists y Lxy)$
- (ii) $\exists x (Px \land \forall y (Py \rightarrow Lxy))$
- (iii) $\forall x (Px \to \forall y (Py \to Lxy))$.

1+2+

(e) Prove that every theorem of Predicate Calculus is logically valid.

5

- (f) Find a derivation for each of the theorems in Predicate Calculus:
 - (i) $\vdash \forall x \forall y \phi \rightarrow \forall y \forall x \phi$

(ii)
$$\vdash \neg \forall x \phi \rightarrow \exists x \neg \phi$$
.

2+3

- (g) (i) When a first order formula is said to have Prenex Normal Form?
 - (ii) Transform the following formula into Prenex Normal Form:

$$\neg (\forall x \exists y \ Pxyz \to \exists x (\neg \forall y \ Qyt \to Rx)).$$
 1+4

Paper: SEC-B-2

[Scientific Computing with SageMath/R]

(Answer either Scientific Computing with SageMath or Scientific Computing with R)

Paper: SEC-B-2.1

(Scientific Computing with SageMath)

Full Marks: 80

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The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Throughout the question paper sage code means code in SageMath.

1. Choose the correct alternative and justify your answer wherever it is applicable: 2×10(a) What will be the output of the following sage code?

17/2-8%3+19//4-2^2

(i) 15/2

(ii) 13/2

(iii) 7.5

(iv) 6.5.

(b) What is the correct code in Sage to find the value of π correct to 10 decimal points?

(i) n(pi, 10)

(ii) n(pi, digits = 10)

(iii) n(pi, digits = 11)

(iv) N(pi, 10).

(c) What will be your code in Sage to evaluate $\sin^{-1}(1/2)$?

(i) arc(sin(1/2))

(ii) asin(1/2)

(iii) aSin(1/2)

(iv) sininv(1/2).

(d) What will be the correct sage command for computing A^{10} , where $A = \begin{bmatrix} 1 & 7 & 1 \\ 1 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}$?

(i) Matrix([[1,7,1], [1,1,0], [3,3,1]])**(10)

(ii) Mat([[1,7,1], [1,1,0], [3,3,1]])**(10)

(iii) $mat([1,7,1;1,1,0;3,3,1])^{(10)}$

(iv) matrix([1,7,1; 1,1,0; 3,3,1])**(10).

- (e) Which one of the following sage commands is correct for computing the integral $I = \int_{-\infty}^{\infty} \frac{1}{(1+x^2)} dx$?
 - (i) integrate(1/(1+x^2);-infinity, infinity)
 - (ii) integrate($1/(1+x^2)$,x,-inf, inf)
 - (iii) integral $(1/(1+x^2), -inf, inf)$
 - (iv) integral $(1/(1+x^2),x,-infinity)$, infinity).
- (f) Which one of the following sage commands is correct for plotting $f(x) = \cos(x)$ in the interval $[-\pi, \pi]$?
 - (i) plot(x, cos(x), -pi, pi)
 - (ii) plot(cos(x), x, -pi, pi)
 - (iii) plot2D(cos(x), x, -pi, pi)
 - (iv) graph(x, cos(x), -pi, pi).
- (g) What will be the output of the following sage code?

(i) 64

(ii) 128

(iii) 32

- (iv) 256.
- (h) What will be the output of the following sage code?

- (i) {2, 3, 4, 5}
- (ii) {1, 3, 4, 5}

(iii) $\{2, 3, 5\}$

- (iv) $\{2, 3, 4\}$.
- (i) Which one of the following sage codes will provide the output 75?
 - (i) S=0 for i in [10, 11,...,15]

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- (iii) S=0 for i in [10, 11, ..., 15]: S = S + i print(S)
- (iv) S=0
 for i in [10, 11, ..., 15]:
 S = S + i
 print(S)
- (j) Which one of the following sage codes will not produce 120 as output?
 - (i) gcd (240, 360)
 - (ii) $f(x) = x^5$ $f(x) \cdot derivative(x, 5)$
 - (iii) $f(x) = x^4$ integrate(f(x), x, 2, 3)
 - (iv) 1cm (12,40)



Section - I

Answer any one question.

- 2. (a) Consider the sage code to find factorial 73. Hence write the sage code to find the number of zeros in factorial 73.
 - (b) Write sage command to find the numerical approximate value of the following expression correct upto 17 digits.

$$\frac{\sqrt{30} + 2^{-.04}}{\left|\sin\frac{\pi}{4} - \tan\frac{\pi}{4}\right|}$$

(c) Give the output of the following sage expression:

$$\exp(I*pi) + e^0 + \log(10^3, 10) - \text{binomial}(4, 2)$$

(d) Write the sage command to calculate all the square roots of (-9).

(1+2)+4+2+1

- 3. (a) Write down the sage command to determine the value of $(55 \sqrt[5]{13})^7$ correct to 7 decimal places.
 - (b) Write the sage commands to compute $\frac{\left[3 + \sqrt[3]{4}\right]^6}{0.01^2} 32$.
 - (c) Write sage commands to find the number of days in 12,345 hours.
 - (d) Write sage commands to find the divisors of the number $3^{33}-1$.
 - (e) Suppose ₹50,000 is invested under compound interest at 5% per annum. Write a sage code to compute the interest amount after 4 years.

 2+2+2+2+2

Section - II

Answer any one question.

- 4. (a) Write sage code to plot the graphs of the curve $f(x) = x x^3$, $x \in [-2, 2]$, it's tangent at the point (1, 0) and a big dot of size 30 at that point in the same plot.
 - (b) Write sage code to plot the function f(x) in the interval [-1, 1] $f(x) = \begin{cases} 1-x, & -1 \le x \le 0 \\ 1+x^3, & 0 < x < 1 \end{cases}$.
- 5. (a) Write sage code to plot the parametric curve: $x = t \sin(1+t^2)$, $y = t \cos(1-t^2)$ for $-2\pi \le t \le 2\pi$.
 - (b) Write sage code to plot the polar curve : $r = 1 + 2\cos 2\theta$, $0 \le \theta \le 2\pi$.
 - (c) Write sage code to plot the function $f(x) = \frac{1}{(x-2)(x-3)}$ in [0, 4] and its asymptotes, if any.
 - (d) Write sage code to plot the graphs of x^2 , e^x and $\sin x$ with different line styles in [-10, 10] in the same plot with the same axes. 2+2+3+3

Section - III

Answer any one question.

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- 6. (a) Write sage codes to find the first three derivatives of $f(t) = \sin(t^2)$ and to plot them together along with the graph of f(t) indicating different colours for different curves.
 - (b) Write the sage command for computing the derivative of $f(x) = \sin(2x + 1)$ at x = 2 from definition of derivative.
- 7. Write sage code to evaluate:

(a)
$$\int e^{-x^3} dx$$

(b)
$$\int_{-1}^{1} x \sin x \, dx$$

(c)
$$\int_{0}^{\infty} \frac{x \cos u}{u^2 + x^2} du$$

(d)
$$\int_{0}^{1} \frac{dt}{\sqrt{t(1-t)}}$$

Section - IV

Answer any one question.

8. (a) Write the sage code to define a matrix M, whose rows are

$$(1, 3, -6, 2), (7, -1, 0, 2), (2, 8, 6, 1), (-5, 9, 0, 1)$$

- (b) Write code in sage to find det M and M^{-1} .
- (c) Express matrix M as the sum of a symmetric and a skew symmetric matrix.
- (d) What will be the output of the sage codes?
 - (i) M[0, 0]
 - (ii) M[2, 3].

2+2+4+2

9. (a) Write a sage code to solve the following system of equations:

$$\begin{cases} x+y+2z=5\\ x+2y+5z=22\\ x+9z=15 \end{cases}$$
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- (b) (i) Write sage codes to input the matrix $A = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{pmatrix}$ and to find the sum of all elements of A.
 - (ii) Also write sage code to find row-reduced echelon form of 4.
- (c) Write sage code to displace a diagonal matrix of order 6 whose all diagonal elements are 11 without entering from keyboard. 4+[(2+1)+2]+1)

Section - V

Answer any one question.

10. (a) Write sage commands to solve the following initial value problem:

$$\frac{dy}{dx} = 1 + xy, y(0) = 1.$$

- (b) Write the sage code to solve the differential equation $\frac{dx}{dt} t^2 \sin t = 0$ and to show the solution. Write also the sage code to plot the solution curves corresponding to the values 1, 2, 3 of arbitrary constant.
- (c) Write the sage codes to solve the polynomial equation $x^3 9x^2 + 2x + 48 = 0$. 2+(2+1+3)+2

11. (a) Write the sage code to solve the following boundary value problem:

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$
; $y(0) = 3$ and $y(\log 2) = 8$

(b) Write the sage code to solve the following initial value problem:

$$\frac{dy}{dx} = e^{x-y}; y(0) = 1$$

- (c) Write the sage code to find the roots of the polynomial equation $x^3 13x^2 121x + 133 = 0$.
- (d) Write sage code to solve the differential equation $\frac{dy}{dx} = \frac{y + \sqrt{y^2 + x^2}}{x}$ 3+3+2+2

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Answer any one question.

- 12. (a) Without using inbuilt functions, write a program in sage to determine the total number of primes less than 100 and to print the list of such primes.
 - (b) Write a program in sage to find the Arithmetic Mean and Geometric Mean of the following data:

- (c) Without using inbuilt functions, write a program in sage to find the factorial of a number n = 10 with user defined sage function. 3+3+4
- 13. (a) Without using inbuilt functions, write a program in sage to find the median of the numbers 9, 11, 7, 32, 14, 7, 28
 - (b) Write a sage program without using any inbuilt function to print first 10 Fibonacci numbers starting with 0, 1.
 - (c) Without using inbuilt functions, write a program in sage to find the LCM of two given numbers.

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Paper: SEC-B-2.2

(Scientific Computing with R)

Full Marks: 80

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

1.		Answer all questions with proper explanation/justification (one mark for correct answer and one mark for justification whenever applicable):							
	(a)	If a	← 10 and b ← 'Hello', what will be	e the	e output of a & b?				
		(i)	True	(ii)	False				
		(iii)	Error	(iv)	None of these.				
	(b)	If A	= 20, B = 25, then how to display ".	20 <	25"? MURALIDHAR GIRLS' COLLEGE				
	1	(i)	print(A, "<", B)	(ii)	paste(A + "<" + B)				
	,)	(iii)	paste(A, "<", B)	(iv)	print(A + "<" + B).				
	(c)								
		(i)	3	(ii)	2				
		(iii)	0	(iv)	None of these.				
	(d)	Whi	ich one of the following is the corre	ct ou	atcome of the command?				
		2+	3-4*6/3-2^3/4+7-6*2**3	/2-	+2 - 4 + 5?				
		(i)	-49	(ii)	-19				
		(iii)	19	(iv)	None of these.				
	(e)	If a	command is not completed in a line	, R v	will give a different prompt, by default it is				
(3	(i)	\$	(ii)	>				
0		(iii)	+	(iv)	%.				
	(f)	f) Which one of the following is the correct command to obtain the multiplication of two square matrices x and y of the same order?							
		(i)	x * y	(ii)	x % * y				
		(iii)	x % * % y	(iv)	x % % * % % y.				

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Sm.)-Mathematics-H/	(12)	
-B-1 & SEC-B-2)/CBCS	c vr 21 for th	e matrix specified by
(g) Which one of the following	is the correct outcome of $X[, 2]$ for the $= 3$, data = $c(10, 20, 30, 40, 50, 60, 70, 10, 10, 10, 10, 10, 10, 10, 10, 10, 1$	80, 90), byrow = F)?
$X \leftarrow \text{matrix } (n\text{row} = 3, n\text{col})$	= 3, data = c(10, 20, 20, 20, 20, 20, 20, 20, 20, 20, 2	

(i) 40 50 60

(ii) 20 50 80

(iii) 10 50 90

- (iv) None of these.
- (h) Which type of graphics R can produce?
 - (i) PDF format

- (ii) PNG format
- (iii) Postscripts format
- (iv) All of these.
- (i) What will be the output of the following R Code snippet? > paste ("a", "b", se = ":")
 - (i) "a + b"

(iii) "ab:"

- (iv) None of these.
- What will be the output of the following R Code?

Options (digits = 6)

(3*7)/11

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(i) 1.90909

(ii) 1.909091

(iii) 1.909

(iv) 1.90.

Section - I

Answer any one question.

- 2. Find the outputs of the following R commands:
 - (a) 1:3 + rep (seq (from = 0, by = 10, to = 30), each = 3)
 - (i) 9%%(7%%3)
 - (ii) (9%%7)%%3
 - (iii) 9%%7%%3
 - (c) $> a \leftarrow \text{matrix} (1:9, n\text{row} = 3)$ $>z \leftarrow c (5, 12, 13)$ > a [z%%2 == 1, 1]
- 3. Find the outputs of the following R codes:
 - (a) $X \leftarrow c("a", "b", "c", "d")$ for (i in 1:4) { print(x[i])

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(13)
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(b) n \leftarrow 0

square \leftarrow 0

while (square < 100) {

+n \leftarrow n+1

+ square \leftarrow n^2

+ }

print (n)

print (square)
```

(c) $> z \leftarrow c(0.2, 1.1, 1.4, 2.5)$ > for $(i \text{ in } z) \{$ + print (2 * i)+ $\}$

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(d) $> X \leftarrow c(0.2, 1.1, 1.4, 2.5)$ > for (i in 1 : length (X)) { + print (2 * X[i])+ }

2+3+2+3

Section - II

Answer any one question.

- 4. Write R code to plot the graph of following in one plot:
 - (a) graph of the function $f(x) = x^3 + x$ in range (-3, 3)
 - (b) graph of the function $f(x) = e^x + x$ in range (-3, 3)
 - (c) graph of the function $f(x) = x^2$ in range (-3, 3).

4+3+3

- 5. Write R code(s) for plotting
 - (a) the graph of polar curve $r = 1 2\cos\theta$, $0 \le \theta \le 2\pi$
 - (b) the following functions in one plot:
 - (i) graph of the function $f(x) = x^2 2$ in range (-3, 3)
 - (ii) graph of the tangent line f'(x) = 2x in range (-3, 3) at the point (2, 2). 4+(3+3)

Section - III

Answer any one question.

- 6. (a) Write R codes to do the following for the function $f(x) = x^2 + e^x$ in $-2 \le x \le 2$:
 - (i) find f'(x)
 - (ii) find f''(x)
 - (iii) plot f'(x) and f''(x) in $-2 \le x \le 2$ with black and blue colors respectively.
 - (b) Compute and plot the graph of the 4th derivative of $cos(2x^2)$ from x = 0 to 5. (1+1+3)+(3+2)

- 7. (a) Write R commands to find the values of the following integrals:
 - (i) $\int_{0}^{\infty} x^4 e^{-x} dx$
 - (ii) $\int_{-1}^{1} \frac{\log(1+x)}{1+x^2} dx$

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- (iii) $\int_{0}^{\frac{\pi}{2}} \sqrt{\tan x} \, dx$
- (b) Write a R program to find roots of the polynomial $x^7 + 3x^4 + 8x + 9$.

(2+2+2)+4

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Section - IV

Answer any one question.

- 8. (a) Write a R program to find factorial of any given number n.
 - (b) Write a R program to find the sum of Fibonacci series up to n numbers.

5+5

5+5

- 9. (a) Write a R program to find the next prime number of a given natural number.
 - (b) Write R program to find the sum of natural numbers up to n using the recursive function.

Section - V

Answer any one question.

- 10. Given the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, write R programs
 - (a) to find A^2 .
 - (b) to find the determinant and the inverse of A, if it exists.
 - (c) select odd elements of A and change those to 0.

2+1+3

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- 11. (a) Write R program to find roots of the polynomial $x^3 3x^2 6x + 8$. Then plot the polynomial and its roots in one graph.
 - (b) Solve the following system of equations by R program:

$$x+y+2z=6$$

 $3x+2y+4z=9$
 $2x+3y-6z=3$
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(2+2+2)+4

Section - VI

Answer any one question.

- 12. (a) Write a R program to evaluate the function $f(x) = 3x^2 + 2x + 1$ in the interval (-1, 1) with steps of 0.2.
 - (b) Write a R code to solve the differential equation with the initial conditions:

$$\frac{dx}{dt} = x(1-x)$$
, where $x(0) = 0.1$. 5+5

- 13. (a) Write a R code to convert Fahrenheit to Centigrade $\left[C = (F-32) \times \frac{5}{9}\right]$. Get your function to print out your result in the following format: 'Fahrenheit: Value of °F' is equivalent to 'Value °C Centigrade'.
 - (b) Write a R program to find the sum of the series:

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$$1^3 + 2^3 + 3^3 + \dots + N^3$$
. (3+2)+5