

2020

## MATHEMATICS — HONOURS

Paper : CC-2

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

1. Choose the correct alternative with proper justification, 1 mark for correct answer and 1 mark for justification : 2×10
- (a) The mapping  $f: \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(n) = n - (-1)^n$ ,  $n \in \mathbb{N}$  is  
 (i) not a 1-1 mapping (ii) not an onto mapping  
 (iii) not a mapping (iv) 1-1 and onto mapping.
- (b) If  $5x + 3 \equiv 5 \pmod{11}$  then one possible value of  $x$  is  
 (i)  $-7$  (ii)  $9$  (iii)  $8$  (iv)  $7$ .
- (c) A relation  $R$  from  $\{1,2,\dots,10\}$  to  $\{1,2,\dots,10\}$  is defined by  $mRn$  if  $m^2 + n^2 = 10$ . Then  $R$  is  
 (i)  $\{(1,3)\}$  (ii)  $\{(3,1)\}$   
 (iii)  $\{(1,3), (3,1)\}$  (iv)  $\{(1,3), (-1,3), (1,-3), (-1,-3)\}$ .
- (d) The range of the function  $f(x) = ([x])^2$ ,  $x \in \mathbb{R}$  is  
 (i)  $\mathbb{N}$  (ii)  $\mathbb{Z}$   
 (iii)  $\{1, 4, 9, \dots\}$  (iv)  $\{0, 1, 4, 9, \dots\}$ .
- (e) The value of  $\beta$  such that the rank of the matrix  $A = \begin{bmatrix} 0 & \alpha & -\alpha \\ \beta & 0 & 0 \\ 1 & -\alpha & \alpha \end{bmatrix}$  ( $\alpha \neq 0$ ) is 2, is  
 (i) 2 (ii) 1 (iii)  $-2$  (iv)  $-1$ .
- (f) If  $\alpha, \beta, \gamma, \delta$  be the roots of the equation  $x^4 + px^3 + qx^2 + rx + s = 0$ , then the value of  $\sum \frac{1}{\alpha^2}$  is  
 (i)  $\frac{2q - r^2}{s}$  (ii)  $\frac{2qs - r^2}{s}$   
 (iii)  $\frac{r^2 - 2q}{s}$  (iv)  $\frac{r^2 - 2qs}{s}$ .

Please Turn Over

(g) The equation whose roots are double of the roots of the equation  $32x^3 - 14x + 3 = 0$  is

(i)  $4x^3 - 7x + 3 = 0$

(ii)  $64x^3 - 28x + 6 = 0$

(iii)  $3x^3 - 7x + 4 = 0$

(iv)  $16x^3 - 7x + 6 = 0.$

(h) The unit digit in  $7^{99}$  is

(i) 7

(ii) 9

(iii) 3

(iv) 1

(i) Consider the set  $A = \left\{ z \in \mathbb{C} : \bar{z} = \frac{1}{z} \right\}$ , then the points of  $A$  lies

(i) on a circle

(ii) on a hyperbola

(iii) on an ellipse

(iv) on a straight line

$$x + y + z = kx$$

(j) The system of equations  $x + y + z = ky$

$$x + y + z = kz$$

will have non-trivial solutions if the values of  $k$  are

(i) 3, 0

(ii) 3, -3

(iii) 0, -3

(iv) 1, 0

2. Answer **any four** questions :

5×4

(a) By Sturm method prove that the roots of the equation  $x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$  are all real.

(b) If  $\alpha, \beta, \gamma$  be the roots of the equation  $x^3 + qx + r = 0$ , find the equation whose roots are  $(\beta - \gamma)^2$ ,  $(\gamma - \alpha)^2$ ,  $(\alpha - \beta)^2$ .

(c) Solve the difference equation  $u_{x+2} + u_{x+1} - 12u_x = 7x$ ,  $x \geq 1$ .

(d) If  $a, b, c$  are positive numbers such that  $a + b + c = 1$ , then show that

$$\sqrt{4a+1} + \sqrt{4b+1} + \sqrt{4c+1} < 5.$$

(e) Solve  $z^8 + z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$  in the field of complex numbers.

(f) If  $\tan(\theta + i\phi) = \sin(\alpha + i\beta)$ , prove that  $\sin 2\theta \cot \alpha = \sinh 2\phi \coth \beta$ .

(g) (i) Apply Descartes' rule of signs to determine the possible nature of the roots of the equation  $x^7 - 3x^3 - x + 1 = 0$ .

(ii) Solve the equation by Ferrari's method :  $x^4 + 3x^3 + 5x^2 + 4x + 2 = 0$ .

1+4

3. Answer **any four** questions :

5×4

(a) (i) Let  $X$  be a non-empty set. Prove that the following conditions are equivalent :

(A)  $\rho$  is an equivalence relation on  $X$ .

(B)  $\rho$  is a reflexive relation on  $X$  and for all  $x, y, z \in X$ , if  $x \rho y$  and  $x \rho z$ , then  $y \rho z$ .

(ii) Let  $R$  be a relation on a set  $A$ . Define  $\tau(R) = R \cup R^{-1} \cup \{(x, x) : x \in A\}$ , show that  $\tau(R)$  is reflexive and symmetric.

3+2

- (b) (i) If  $f: \mathbb{N} \rightarrow \mathbb{N}$  and  $g: \mathbb{N} \rightarrow \mathbb{N}$  defined by  $f(x) = x + 1$  and  $g(x) = \max.\{0, x - 1\}$  for  $x \in \mathbb{N}$ , then show that  $gof = I_{\mathbb{N}}$  but  $fog \neq I_{\mathbb{N}}$  where  $I_{\mathbb{N}}$  is an identity function on  $\mathbb{N}$ .
- (ii) If the function  $f: \mathbb{Z}_5 \rightarrow \mathbb{Z}_5$  is defined by  $f(x) = 2x$  for all  $x \in \mathbb{Z}_5$ , then find  $f^{-1}([3])$ , where  $\mathbb{Z}_5$  is the set of all equivalence classes on  $\mathbb{Z}$  corresponding to the equivalence relation modulo 5. 3+2
- (c) Let  $P = \{x \in \mathbb{R} : 0 < x < 1\}$  and  $f: P \rightarrow \mathbb{R}$  be defined by  $f(x) = \frac{2x-1}{1-|2x-1|}$ . Is  $f$  bijective? Justify. If so, find  $f^{-1}$ . 3+2
- (d) (i) If  $p$  be prime and  $k$  be a (+ve) integer, then prove that  $\phi(p^k) = p^k \left(1 - \frac{1}{p}\right)$ .
- (ii) If  $a$  is relatively prime to  $b$ , prove that  $a^2$  is also relatively prime to  $b$ . 3+2
- (e) Let  $P$  be the set of all positive divisors of 36. On  $P$  define a relation  $\rho$  by : for  $a, b \in P$ ,  $a\rho b$  if and only if  $a | b$ . Prove that  $(P, \rho)$  is a poset. Is  $(P, \rho)$  a linear ordered set? Justify your answer. 3+2
- (f) If  $p$  is a prime number such that  $p \geq 5$ , then prove that  $p^2 - 1$  is divisible by 24. 5
- (g) Using Chinese remainder theorem solve the following system of congruence equations 5

$$2x \equiv 1 \pmod{3}$$

$$5x \equiv 4 \pmod{4}.$$

4. Answer **any one** question :

5×1

- (a) Check the consistency of the system of equations

$$2x - y + z = 4$$

$$3x - y + z = 6$$

$$4x - y + 2z = 7$$

$$-x + y - z = 9.$$

- (b) Reduce the following matrix in the row reduced echelon form :

$$\begin{bmatrix} 1 & 3 & 0 & 5 & 2 \\ 0 & 0 & 3 & 4 & 0 \\ 7 & 1 & 0 & 4 & 1 \\ 5 & 3 & 2 & 1 & 6 \end{bmatrix}.$$


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