

2020

MATHEMATICS — HONOURS

Paper : CC-7

Full Marks : 65

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.* \mathbb{R} denotes the set of real number

Group - A

(Marks : 20)

1. Answer the following multiple choice questions with only one correct option. Choose the correct option and justify ; (1+1)×10
- (a) If x, x^2, x^3 are three linearly independent solutions of a third-order differential equation, then the Wronskian W of the functions has value
 (i) $W = 2x^3$ (ii) $W = x^3$ (iii) $W = x^2$ (iv) $W = 2x^2$.
- (b) One of the points which lies on the solution curve of the differential equation $(y-x)dx + (x+y)dy = 0$ with given condition $y(0) = 1$ is
 (i) $(1, -2)$ (ii) $(2, -1)$ (iii) $(2, 1)$ (iv) $(-1, 2)$.
- (c) If the integrating factor of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^m y^n$, then
 (i) $m = -7, n = 1$ (ii) $m = 1, n = -7$ (iii) $m = 0, n = 0$ (iv) $m = 1, n = 1$.
- (d) Let $y(x)$ be the solution of the differential equation $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, y(0) = 1, \left. \frac{dy}{dx} \right|_{x=0} = -1$, then $y(x)$ attains its maximum value at
 (i) $\ln \frac{4}{3}$ (ii) $\ln \frac{3}{4}$ (iii) $\ln \frac{1}{2}$ (iv) none of these.
- (e) Consider the differential equation $a\frac{dy}{dx} + by = ce^{-\lambda x}$, where a, b, c are positive constants and λ is a non-negative constant. Then every solution of the differential equation approaches to $\frac{c}{b}$ as $x \rightarrow +\infty$ when
 (i) $\lambda > 0$ (ii) $\lambda = 0$ (iii) $\lambda = \frac{b}{a}$ (iv) $\lambda = \frac{a}{b}$.

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3. Reduce the equation $x^3p^2 + x^2yp + a^3 = 0$ to Clairaut's form by the substitution $y = u$ and $x = \frac{1}{v}$ and obtain the complete primitive.

4. Solve using the method of undetermined coefficients, the equation with initial conditions,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x, \quad y(0) = 2 \text{ and } y'(0) = 4.$$

5. Solve by the method of variation of parameters the equation $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = \log x, (x > 0)$.

6. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos \log(1+x)$ by changing the independent variables.

7. Use D-operator to solve :

$$\frac{d^2y}{dx^2} - y = x \sin x + (1+x^2)e^x$$

8. Show that the equation of the curve, whose slope at any point (x, y) is equal to $xy(x^2y^2 - 1)$ and which passes through the point $(0, 1)$ is $x^2y^2 = 1 - y^2$.

9. Solve for x from the system of equations

$$\begin{aligned} \frac{dx}{dt} + 4x + 3y &= t \\ \frac{dy}{dt} + 2x + 5y &= e^t \end{aligned}$$

10. Consider the plane autonomous system

$$\frac{dx}{dt} = 2x + y, \quad \frac{dy}{dt} = 3x + 4y$$

Find the general solution of the system. State the nature of the critical point of the system. Discuss its stability. Draw a phase portrait of the system.

11. Solve the equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$ in series about the ordinary point $x = 0$.

Group - C
(Marks : 15)

Answer *any three* questions.

12. Define limit point of a subset of $\mathbb{R} \times \mathbb{R}$. If $B = \{(a, 0); a \in \mathbb{R}\}$. Show that B is a closed set but not open in $\mathbb{R} \times \mathbb{R}$. 5

13. State the sufficient conditions for differentiability of a function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$. Examine whether the sufficient conditions of differentiability are satisfied for the following function $f(x, y)$ and hence comment

on differentiability of $f(x, y)$ at $(0, 0)$ where $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2} & , \text{ when } x^2 + y^2 \neq 0 \\ 0 & , \text{ when } x^2 + y^2 = 0. \end{cases}$ 1+4

14. If z is a function of two variables x and y and $x = c \cosh u \cos v$, $y = c \sinh u \sin v$ (c is a real number), show that

$$\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = \frac{c^2}{2} (\cosh 2u - \cos 2v) \left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} \right). \quad 5$$

15. Find all critical points of the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = x^3 + y^3 - 3x - 12y + 40$ for $(x, y) \in \mathbb{R}^2$. Also examine whether the function f attains a local maximum or a local minimum at each of these critical points. 5

16. Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \text{ by the method of Lagrange's multipliers.} \quad 5$$