2022

MATHEMATICS — HONOURS

Paper : DSE-B-1.2

(Linear Programming and Game Theory)

Full Marks : 65

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***all*** questions with proper explanation/justification (**one** mark for correct answer and **one** mark for justification) : 2×10

(a) For the following L.P.P. :

Maximize $Z = 2x_1 + 3x_2$

subject to $3x_1 + x_2 \le 3,$

$x_1 \ge 0, x_2 \ge 0,$

the optimal value of the objective function will be

(i) 2 (ii) 9

(iii) 3 (iv) 1.

(b) Which one of the following is a convex set with infinite number of extreme points?

(i) $\{(x_1, x_2) : x_1^2 + x_2^2 = 16\}$ (ii) $\{(x_1, x_2) : x_1^2 + x_2^2 < 16\}$

(iii) $\{(x_1, x_2) : x_1^2 + x_2^2 \le 16\}$ (iv) $\{(x_1, x_2) : x_1^2 + x_2^2 > 16\}$.

(c) The intersection of two convex sets

(i) may or may not be a convex set. (ii) is not a convex set.

(iii) is a convex set. (iv) none of (i), (ii) and (iii).

(d) The following L.P.P. :

Maximize $Z = x_1 + x_2$

subject to $x_1 + x_2 \le 1,$

$-3x_1 + x_2 \ge 3,$

$x_1 \ge 0, x_2 \ge 0,$

has

(i) one feasible solution (ii) two feasible solutions

(iii) infinite number of feasible solutions (iv) no feasible solution.

(e) The minimum cost for the transportation

	D_1	D_2	
O_1	1	2	5
O_2	3	2	10
	8	7	

is

(i) 32 (ii) 30

(iii) 28 (iv) 25

(f) If the primal problem has m constraints with n variables, then the dual problem has

(i) m constraints with n variables.

(ii) n constraints with m variables.

(iii) $m+n$ constraints with mn variables.

(iv) mn constraints with $m+n$ variables.

(g) In 2×2 pay-off matrix $\begin{pmatrix} 6 & 1 \\ x & 5 \end{pmatrix}$ the value of the game is 5. Then the value of x is

(i) 0 (ii) 1

(iii) 3 (iv) 5

(h) In a simplex method, if the optimality condition be satisfied at any iteration and the basis contains one or more artificial vectors with corresponding artificial variable at zero level, then the solution obtained is

(i) optimal (ii) not feasible

(iii) feasible but not optimal (iv) feasible but not basic.

(i) Suppose in a travelling salesman problem, the salesman has to visit n cities. Then the number of possible routes are

(i) n (ii) $n-1$

(iii) $\lfloor n$ (iv) $\lfloor(n-1)$

(j) For the system of equations

$x_1+2x_2+x_3=3,$

$2x_1+x_2+5x_3=9,$

the solution $x_1=5,\ x_2=-1,\ x_3=0$ is

(i) basic and feasible (ii) feasible but not basic

(iii) degenerate basic (iv) non-degenerate basic.

Unit - I

2. Answer *any two* questions :

(a) Three different types of lorries A, B and C have been used to transport 60 tons solid and 35 tons liquid substance. A type lorry can carry 7 tons solid and 3 tons liquid. B type lorry can carry 6 tons solid and 2 tons liquid and C type lorry can carry 3 tons solid and 4 tons liquid. The cost of transport are Rs. 500, Rs. 400 and Rs. 450 per lorry of A, B, and C type respectively. Formulate the problem in L.P.P. form to find the minimum transport cost. 5

(b) $x_1 = 1, x_2 = 1, x_3 = 1$ and $x_4 = 0$ is a feasible solution of the system of equations

$$x_1 + 2x_2 + 4x_3 + x_4 = 7,$$
$$2x_1 - x_2 + 3x_3 - 2x_4 = 4.$$

Reduce the feasible solution to two different basic feasible solutions. 3+2

(c) Define convex polyhedron and convex hull. Give an example of each of them.

Check whether the set

$$S = \{(x, y) : 7x + 8y \le 56, 4x + 5y \le 20, 2x + 3y = 6\}$$

is convex or not. 2+3

(d) State the fundamental theorem of L.P.P. Prove that the set of all feasible solutions to an L.P.P. $AX = b$, $X \ge 0$, is a closed convex set. 2+3

Unit - II

3. Answer *any one* question :

(a) (i) Apply simplex method to solve the following L.P.P. :

Maximize $z = 5x_1 + 3x_2$

subject to $3x_1 + 5x_2 \le 15,$

$5x_1 + 2x_2 \le 10,$

$x_1, x_2 \ge 0.$

(ii) Solve the following L.P.P. by penalty method :

Maximize $z = -2x_1 + x_2 + 3x_3$

subject to $x_1 - 2x_2 + 3x_3 = 2,$

$3x_1 + 2x_2 + 4x_3 = 1,$

$x_1, x_2, x_3 \ge 0.$ 5+5

(b) (i) Use two-phase method to solve the following L.P.P. : 6

Minimize $z = x_1 + x_2 + x_3$

subject to $x_1 - 3x_2 + 4x_3 = 5,$

$x_1 - 2x_2 \le 3,$

$2x_2 + x_3 \ge 4,$

$x_1, x_2, x_3 \ge 0.$

(ii) Show by simplex method that the following problem has an unbounded optimal solution : 4

Maximize $Z = 2x_1 + x_2$

subject to $x_1 - x_2 \le 10,$

$2x_1 - x_2 \le 40,$

$x_1,\ x_2 \ge 0.$

Unit - III

4. Answer *any one* question :

(a) (i) If X be any feasible solution to primal problem and V be any feasible solution to its dual problem, then prove that $CX \le b'V$, where C and b are respectively the cost vector and the requirement vector of the primal problem.

(ii) Formulate the dual of the L.P.P. given below :

Maximize $Z = 2x_1 + 3x_2 + 4x_3$

subject to $x_1 - 5x_2 + 3x_3 = 7,$

$2x_1 - 5x_2 \le 3,$

$3x_2 - x_3 \ge 5,$

$x_1,\ x_2 \ge 0,$

and x_3 is unrestricted in sign. 5+5

(b) Use duality to find the optimal solution (if any) of the following L.P.P. :

Minimize $Z = x_1 + 2x_2 + 3x_3$

subject to $x_1 - x_2 + x_3 \ge 4,$

$x_1 + x_2 + 2x_3 \le 8,$

$x_2 - x_3 \ge 2,$

$x_1,\ x_2,\ x_3 \ge 0.$ 10

Unit - IV

5. Answer *any three* questions :

(a) Solve the following transportation problem : 5

	D_1	D_2	D_3	D_4	$a_i \downarrow$
O_1	10	20	5	7	15
O_2	18	09	12	8	25
O_3	15	14	16	18	5
$b_j \rightarrow$	5	15	15	10	

(b) Solve the following travelling salesman problem so as to minimize the cost per cycle :

To

From	1	2	3	4	5
1	∞	6	12	6	4
2	6	∞	10	5	4
3	8	7	∞	5	3
4	5	4	10	∞	5
5	5	2	7	8	∞

5

(c) Let $f(x, y)$ be a real valued function of x and y defined for $x \in A$ and $y \in B$, A and B being two subsets of real numbers. Now if both $\max\limits_{x\in A}.\min\limits_{y\in B}.f(x, y)$ and $\min\limits_{y\in B}.\max\limits_{x\in A}.f(x, y)$ exist, then prove that $\max\limits_{x\in A}.\min\limits_{y\in B}.f(x, y) \le \min\limits_{y\in B}.\max\limits_{x\in A}.f(x, y)$ 5

(d) In a rectangular game, the payoff matrix A is given by

$$A = \begin{pmatrix} 3 & 2 & -1 \\ 4 & 0 & 5 \\ -1 & 3 & -2 \end{pmatrix}$$

Give reasons whether the players will use pure or mixed strategies. Also find the value of the game. 5

(e) Use dominance to reduce the pay-off matrix and then solve the game problem : 5

Player B

Player A				
	−5	3	1	20
	5	5	4	6
	−4	−2	0	−5

(f) Solve the following game graphically : 5

Player B

Player A	B_1	B_2
A_1	1	2
A_2	5	4
A_3	−7	9
A_4	−4	−3
A_5	2	1