2021

MATHEMATICS — HONOURS

Paper : DSE-B(2)-2

(Astronomy and Space Science)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations have usual meanings.

Group - A

1. Answer all the following multiple choice questions. For each question **1** mark for choosing correct option and **1** mark for justification. 2×10

(a) A telescope observing in space at a wavelength of 800 nm has an aperture with a diameter of 5 m. What is its angular resolution?

(i) 1.95×10^{-7} arcsec (ii) 4.03×10^{-2} arcsec

(iii) 1.95×10^{-1} arcsec (iv) 1.6 arcsec.

(b) A star of magnitude +4 lies at a distance of 100 *pc*. Then the absolute magnitude of the star is

(i) + 9.0 (ii) + 4.0

(iii) + 1.49 (iv) – 1.0.

(c) The redshift of a nearby galaxy is 0.01. If the Hubble constant is 73 km s^{-1} Mpc^{-1}, then the distance of the galaxy in Mpc is

(i) 7.3 Mpc (ii) 21.9 Mpc

(iii) 41.1 Mpc (iv) 730 Mpc.

(d) The microwave background radiation has a spectrum which peaks at a wavelength of 1.1 mm and is identical in shape to that of a black body of temperature 2.7 K. At what wavelength will the spectrum of the star Sirius A (with temperature 9940 K) peak?

(i) 9036 nm (ii) 335 nm

(iii) 299 nm (iv) 34 nm.

(e) The sun will spend 1.1×10^{10} yr on the main sequence. Given that the main sequence stars obey a mass luminosity relationship of the form $L \propto M^{3.5}$. What is the lifetime of a $3M_{\odot}$ star? ($M_{\odot}$ represents solar mass)

(i) 1.08×10^{8} yr (ii) 9.05×10^{8} yr

(iii) 2.13×10^{8} yr (iv) 6.9×10^{8} yr.

(f) A star has a parallax of 0.01 *arcseconds*. Then the distance of the star will be

(i) 3.26 light years (ii) 326 light years

(iii) 100 light years (iv) 10 light years.

(g) The distance of the Sun from the centre of our galaxy is 8.5 *kpc*. What will be the circular velocity of the Sun around the galactic centre?

[Take the constants $A = 14.4$ km s^{-1} kpc^{-1} and $B = -12$ km s^{-1} kpc^{-1}]

(i) 250 km s^{-1} (ii) 224.4 km s^{-1}

(iii) 242.2 km s^{-1} (iv) 220.1 km s^{-1}.

(h) Suppose we look at two distant galaxies : Galaxy 1 is twice as far away as Galaxy 2. In that case

(i) We are seeing Galaxy 1 as it looked at an earlier time in the history of the universe than Galaxy 2

(ii) We are seeing Galaxy 1 as it looked at a later time in the history of the universe than Galaxy 2

(iii) Galaxy 1 must be twice as big as Galaxy 2

(iv) Galaxy 2 must be twice as old as Galaxy 1.

(i) The dimensions of the Reynold's number is

(i) $[M^2L^3T]$ (ii) $[ML^3T]$

(iii) $[M^2L^2T^2]$ (iv) None of these.

(j) The expansion of the universe will be halted if the mass density of the Universe be equal to the critical density ρ_c whose value is [Take $H = 70$ km s^{-1} Mpc^{-1}]

(i) 0.5×10^{-29} gm cm^{-3} (ii) 1×10^{-29} gm cm^{-3}

(iii) 1.5×10^{-29} gm cm^{-3} (iv) 2×10^{-29} gm cm^{-3}.

Group - B

2. Answer ***any one*** question : 5×1

(a) In connection with the spherical triangle, given the observer's latitude 'φ', the declination 'δ' and hour angle 'H' of the heavenly body, calculate its zenith distance and azimuth. Also given the observer's latitude 'φ', the star's zenith distance 'z' and azimuth 'A', calculate the star's declination and hour angle. 3+2

(b) Derive the fundamental formula of spherical trigonometry. 5

Group - C

3. Answer ***any one*** question : 5×1

(a) Discuss the different layers of Earth's atmosphere, indicating the major constituents and their interaction with electromagnetic radiation of different wavelengths. 5

(b) What is f/a ratio of a telescope and what are its various advantages? Compare the brightness of images of the Moon produced by two telescopes – one with $f = 200$ cm, $a = 40$ cm, and the other with $f = 600$ cm and $a = 100$ cm. 2+3

Group - D

4. Answer ***any two*** questions : 5×2

(a) Define luminosity of a star. What is its relation with the effective temperature of a star? Derive the relationship between the luminosity and the absolute magnitude of a star. 1+1+3

(b) What is stellar parallax? The apparent magnitude of a star is observed to be +3.3 and its parallax is $0''.025$. Find the absolute magnitude of the star. Compare the luminosity of this star with that of the Sun ($M_{v\odot} = +5.0$). 1+2+2

(c) The coronal spectrum shows emission lines of intense ionization— Explain. Comment on the sources of the coronal heating. 3+2

(d) Discuss the solar neutrino puzzle and its possible solutions. 5

Group - E

5. Answer ***any one*** question : 5×1

(a) What are interstellar shock waves? Write down the equations which are appropriate for studying the propagation of a plane, normal and adiabatic shock. Deduce the Rankine-Hugoniot relation. 1+2+2

(b) Define Jeans wavelength, λ_j and Jeans Mass M_j. How are they related to the gravitational collapse of a static homogeneous cloud? Derive expressions for them. 1+1+3

Group - F

6. Answer ***any two*** questions : 5×2

(a) Derive the formulae for the radial velocity, v_r and the tangential velocity, v_T in terms of the Oort's constants A and B. 5

(b) Draw a diagram of the rotation curve of our galaxy and obtain a polynomial in the radial distance 'r' that fits the rotation curve fairly well. 2+3

(c) Describe Hubble's morphological classification of galaxies. What are the principal observable features that form the basis for this classification? What features distinguish the sub-classes? 2+2+1

(d) Discuss the observations that suggest that a very large fraction of matter remains hidden in individual galaxies, galaxy clusters and in the universe. Also derive an estimate of the hidden matter. 3+2

Group - G

7. Answer ***any two*** questions : 5×2

(a) If 'm_0' and 'm_f' are respectively the initial and final mass of a rocket, then prove that $m_f = m_0 \exp\left(-\frac{\Delta v}{c}\right)$, where Δv is the difference between the initial and final velocity of the rocket and 'c' is the velocity of exhaust. 5

(b) As an approximate of Navier–Stokes equation of motion, derive the boundary layer equations for two-dimensional incompressible fluid flow past a flat plate. 5

(c) What is Blasius boundary layer flow? Deduce the self-similar equation for this flow. 1+4

(d) Write a note on the remarkable achievements of the Indian Space Research Organization (ISRO). 5

[Throughout the Paper take the Newton's Gravitational constant as $G = 6.67\times10^{-11}\ m^3kg^{-1}s^{-2}$].