2021

MATHEMATICS — HONOURS

Fifth Paper

(Module - X)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

(Notations and symbols have their usual meaning.)

Group-A

(Marks-20)

Section-I

(Linear Algebra-II)

Answer ***any one*** question. 10×1

1. (a) Let V and W be vector spaces of finite dimension over a field F and $T : V \to W$ be a linear mapping. Then show that the rank of T = the rank of matrix T. 5

(b) A mapping $F : \mathbb{R}^3 \to \mathbb{R}^3$ maps the vector (2, 1, 1), (1, 2, 1) and (1, 1, 2) to (1, 1, –1), (1, –1, 1) and (1, 0, 0) respectively. Show that F is not an isomorphism. 5

2. (a) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^4$ is defined by $T(x, y, z) = (y + z, z + x, x + y, x + y + z), (x, y, z) \in \mathbb{R}^3$. Find Im T and dimension of Im T. 5

(b) A linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by $T(0,1,1) = (1,0,1)$, $T(1,0,1) = (2,3,4)$, $T(1,1,0) = (1,2,3)$. Find the matrix of T relative to the order basis $(\epsilon_1, \epsilon_2, \epsilon_3)$ where $\epsilon_1 = (1,0,0)$, $\epsilon_2 = (0,1,0)$, $\epsilon_3 = (0,0,1)$. Deduce that T is invertible. 5

Section-II

(Modern Algebra-III)

Answer ***any one*** question. 10×1

3. (a) Prove or disprove : A subgroup H of a group G is a normal subgroup if and only if every right coset of H is also a left coset. 4

(b) Let G be a group. Let H be a subgroup of G such that $H \subseteq Z(G)$. Show that if G/H is cyclic then $G = Z(G)$, where $Z(G)$ denotes the centre of G. 3

(c) Prove that the quotient group $(Q/Z, +)$ is infinite but each of its elements is of finite order. 3

4. (a) Suppose that there is a homomorphism from a group G on Z_{10}. Prove that G has normal subgroups of index 2 and 5. 3

(b) If K is a subgroup of G and N is a normal subgroup of G, prove that $K/(K \cap N)$ is isomorphic to KN/N. 3

(c) Find all homomorphism from Z_6 into Z_4. How many of those are epimorphism? Justify your answer. 2+2

Group-B

(Tensor Calculus)

(Marks-15)

Answer ***any three*** questions. 5×3

5. If $A^i (i = 1,2, \dots, n)$ are components of an arbitrary contravariant vector and $C_{ij}A^iA^j$ is an invariant then prove that $C_{ij} + C_{ji} (i, j = 1,2 \dots, n)$ are components of a second order tensor of type (0, 2). 5

6. If $A_{ij} (i, j = 1,2 \dots, n)$ are components of a skew-symmetric tensor of rank 2, then prove that

$$\left(\delta_j^i \, \delta_l^k + \delta_l^i \, \delta_j^k\right) A_{ik} = 0.$$ 5

7. If all the components of a tensor are zero at a point in one co-ordinate system, then prove that they are all zero at this point in every co-ordinate system. 5

8. If A^{ij} is a skew-symmetric tensor, then show that

$$A^{ij}{}_{,j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} \left(\sqrt{g} \, A^{ij}\right).$$ 5

9. Find g and g^{ij} corresponding to the line element

$ds^2 = 3(dx^1)^2 + 2(dx^2)^2 + 4(dx^3)^2 - 6(dx^1)\,(dx^3)$ in Riemannian space V_3. 5

Answer either **Group-C** or **Group-D.**

Group-C

(Differential Equation-II)

(Marks-15)

Answer ***any one*** question. 15×1

10. (a) State the first shifting property of Laplace transformation. Using this property, find the Laplace transform of $e^{-2t}(3\cos 6t - 5\sin 6t)$. 1+4

(b) Find the inverse Laplace transform of $\frac{4p+5}{(p-1)^2(p+2)}$. 5

(c) Find the power series solution of the initial value problem

$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 2y = 0,\ y(0) = 1,\ y^1(0) = 0.$ 5

11. (a) Find the Laplace transform of $t^2 e^{at} \sin at$. 5

(b) Using shifting property of Inverse Laplace Transform, evaluate $L^{-1}\left\{\frac{6p-4}{p^2-4p+20}\right\}$. 5

(c) Solve by using Laplace transform of $\frac{d^2y}{dt^2} + 9y = \cos 2t$, when $y(0) = 1$ and $y\left(\frac{\pi}{2}\right) = -1$. 5

Group-D

(Graph Theory)

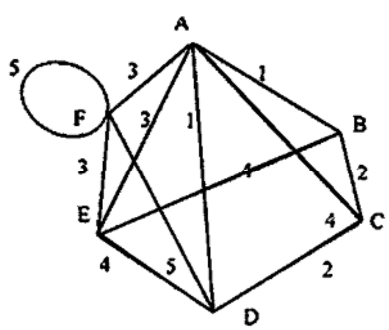
(Marks-15)

Answer ***any three*** questions. 5×3

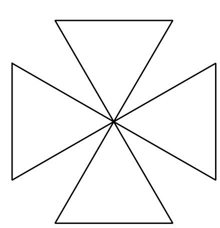
12. (a) Show that there is no simple graph with six vertices of which the degrees of five vertices are 5, 5, 3, 2 and 1. 2

(b) Prove that the number of odd degree vertices of a graph G is always even. 3

13. Obtain a minimal spanning tree of the following graph using Kruskal's algorithm. 5



14. (a) Find a Euler trail in the following graph G. 3



(b) Explain spanning tree in a simple connected graph with example. 2

Please Turn Over

15. State and prove the necessary and sufficient condition for a graph to be an Euler graph. 5

16. (a) Show that a complete graph with *n* vertices consists of $^{n(n-1)}/_{2}$ edges. 3

(b) Prove that a connected graph with *n* vertices and $(n-1)$ edges is a tree. 2
