2020

MATHEMATICS — HONOURS

Paper : DSE-B-1

(Discrete Mathematics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer the following multiple choice questions (M.C.Q.) in which only one option is correct. Choose the correct option with proper justification if any. 2×10

(a) Let G be an undirected 3-regular graph on 10 vertices. Then the number of edges of G is

(i) 13 (ii) 14 (iii) 15 (iv) 16.

(b) The maximum degree of any vertex in a simple graph with n-vertices is

(i) $n-1$ (ii) $n+1$ (iii) $2n-1$ (iv) n.

(c) The maximum number of edges of a connected simple graph with n vertices is

(i) $2.{}^nC_2$ (ii) ${}^nC_2$ (iii) $n-1$ (iv) None of these.

(d) A connected graph has 15 vertices and 20 edges. Then the least number of edges to be removed from the graph to make it a tree is

(i) 13 (ii) 5 (iii) 19 (iv) 6.

(e) Any tree with $n \geq 3$ vertices has at least

(i) one pendant vertex (ii) two pendant vertices

(iii) no pendant vertex (iv) three pendant vertices.

(f) The greatest and least elements of the poset $(P(S), \subseteq)$ with $S = \{a, b, c\}$ are respectively :

(i) $\{a,b,c\}$ and $\{a\}$ (ii) S and ϕ (iii) S and $\{a, b\}$ (iv) None of these.

(g) $n^7 - n$ is divisible by m for all $n \in \mathbb{N}$. Then the value of m cannot be

(i) 14 (ii) 21 (iii) 28 (iv) 42.

(h) What is $\tau(180)$ =

(i) 15 (ii) 16 (iii) 17 (iv) 18.

(i) Check which of the following number is not a Mersenne number?

(i) 1023 (ii) 2049 (iii) 8191 (iv) None of these.

(j) What is the remainder when 5^{48} is divided by 12?

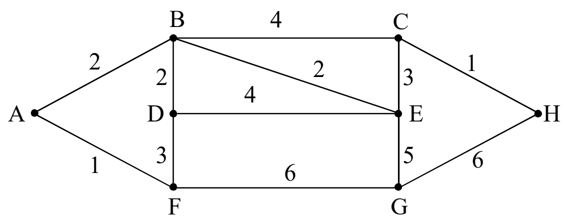
(i) 1 (ii) 2 (iii) 3 (iv) 4.

Unit - 1

Answer ***any five*** questions.

2. Prove that a simple graph with n-vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. 5

3. Use Dijkstra's algorithm to find the shortest path between the vertices A and H in the weighted graph:



5

4. Show that a tree T with n number of vertices has $(n-1)$ edges. 5

5. Let G = (V, E) be a connected graph. Show that $n-e+f=2$, where n, e and f are the number of vertices, edges and regions respectively of the graph. 5

6. If G is a connected planar graph with $n(\geq 3)$ vertices and e edges, then prove that $e \leq 3n-6$. Also prove that the converse of the result is not always true. 3+2

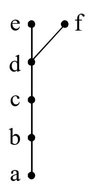
7. (a) Show that a connected graph is Eulerian if every vertex has even degree. $(4 \leq n \leq 20)$.

(b) For which values of n, the complete graph K_n is an Eulerian graph? 3+2

8. Let $X = \{x_1, x_2, ..., x_{100}\}$ be a set of 100 distinct positive integers. If these positive integers are divided by 75, then show that at least two of the remainders must be the same. 5

9. (a) When a partially ordered set $(L, \leq)$ is said to be a lattice? Give example.

(b) Determine whether the poset P represented by the Hasse diagram in the figure given below is a lattice. 2+3



Unit - 2

Answer ***any four*** questions.

10. (a) Use Fermat's theorem to prove that :

$1^{p-1}+2^{p-1}+3^{p-1}+\ldots+(p-1)^{p-1} \equiv (-1)$ mod p, when p is an odd prime.

(b) If p is prime then $(a+b)^p \equiv \left(a^p+b^p\right)(\operatorname{mod} p), \forall a, b \in Z$. 3+2

11. Find the solution of the following system of equations, with help of Chinese Remainder Theorem : $x \equiv 2(\operatorname{mod} 4),\ x \equiv 3(\operatorname{mod} 7),\ x \equiv 2(\operatorname{mod} 9)$. 5

12. Let q be an odd prime and $p = 4q + 1$ be also a prime. Prove that the congruence $x^2 \equiv -1(\operatorname{mod} p)$ has exactly two solutions, each of which is quadratic non-residue of p. 5

13. Let p be an odd prime. Show that any prime divisor of the Mersenne number M_P is of the form $1+2kp$, $k \in \mathbb{N}$. Hence deduce that the number of primes is infinite. 4+1

14. State when a positive integer $n>1$ is said to be perfect. Give an example. Prove that if 2^k-1 is prime $(k>1)$, then $n=2^{k-1}\left(2^k-1\right)$ is perfect for $k>1$. 1+1+3

15. Solve the quadratic congruence $3x^2 + 9x + 7 \equiv 0$ (mod 13). 5

16. (a) Prove Euler Totient function ϕ satisfies $\phi(mn)=\phi(m)\phi(n)$ where gcd (m, n) = 1.

(b) Find $\phi(2520)$. 3+2