2020

MATHEMATICS — GENERAL

Paper : DSE-A-2

(Graph Theory)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Day 3

1. Choose the correct answer(s) from the given options : 1×10

(a) Which of the following statement for a simple graph is correct?

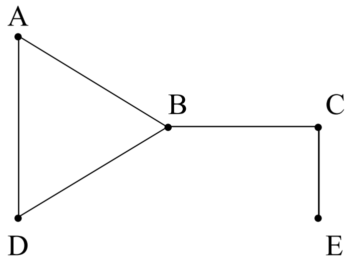
(i) Every path is a trail.

(ii) Every trail is a path.

(iii) Every path is a trail as well as every trail is a path.

(iv) Path and trail have no relation.

(b) In the given graph identify the cut vertices.



(i) B and E (ii) C and D

(iii) A and E (iv) C and B

(c) A connected planar graph having 6 vertices, 7 edges contains ________ regions.

(i) 15 (ii) 3

(iii) 1 (iv) 11

(d) What is the number of edges present in a complete graph having n vertices?

(i) $\frac{n\ (n+1)}{2}$ (ii) $\frac{n\ (n-1)}{2}$

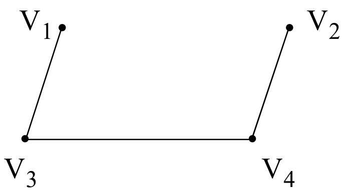
(iii) n (iv) None of the above.

(e) What is the maximum number of edges in a bipartite graph having 10 vertices?

(i) 24 (ii) 21

(iii) 25 (iv) 16.

(f) What would be the number of zeros in the adjacency matrix of the given graph?



(i) 10 (ii) 6

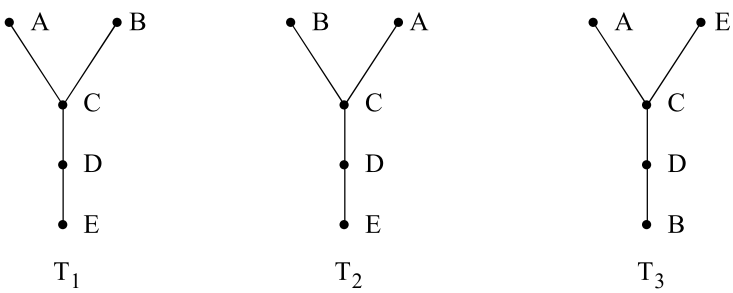
(iii) 16 (iv) 0.

(g) Which of these adjacency matrices represents a simple graph?

(i) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

(iii) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$.

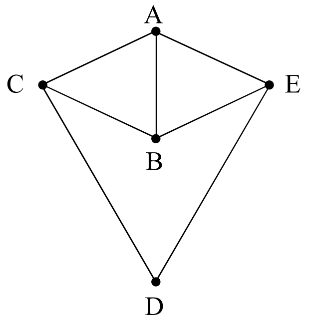
(h) Among the following three trees T_1, T_2 and T_3



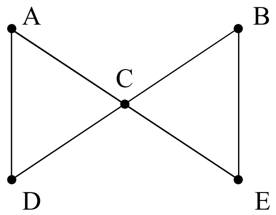
(i) T_1 and T_2 are isomorphic. (ii) T_1 and T_3 are isomorphic.

(iii) T_2 and T_3 are isomorphic. (iv) T_1 , T_2 , T_3 are isomorphic.

(i) For the following two graphs G_1 and G_2



G_1



G_2

(i) G_2 is Hamiltonian but G_1 is not. (ii) G_1 is Hamiltonian but G_2 is not.

(iii) Both G_1 and G_2 are Hamiltonian. (iv) None of G_1 and G_2 is Hamiltonian.

(j) Dijkstra's Algorithm is applicable to the weighted graph with

(i) positive weights only. (ii) both positive and negative weights.

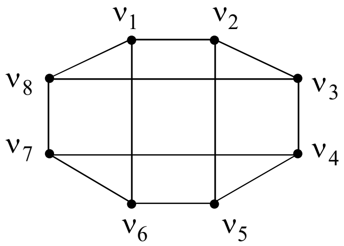
(iii) negative weights only. (iv) none of the above.

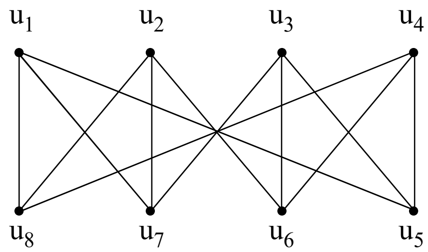
2. Answer ***any three*** of the following questions :

(a) Draw the graph whose adjacency matrix is given by 5

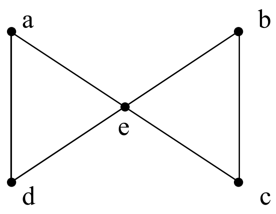
$$\begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) Are the following two graphs isomorphic? Justify your answer. 5



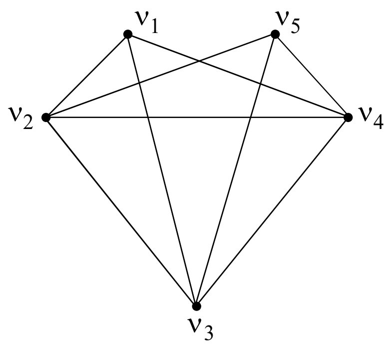


(c) Draw all the spanning trees of the graph : 5



Please Turn Over

(d) What is a planar graph? Is the following graph planar? Justify your answer. 1+4



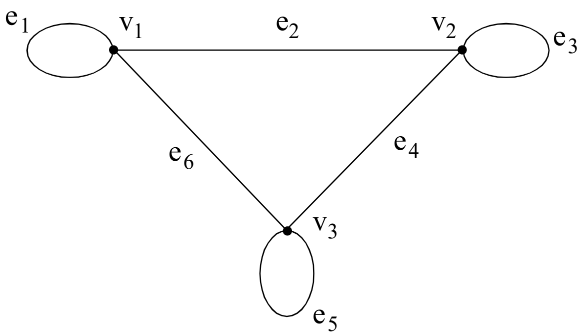
(e) Show that a graph with n vertices, $(n - 1)$ edges and no circuits is connected. 5

3. Answer ***any four*** questions :

(a) (i) Show that there is no simple graph with six vertices of which the degrees of five vertices are 5, 5, 3, 2 and 1.

(ii) Prove that a connected graph with n vertices is a tree if and only if it has $(n - 1)$ edges.

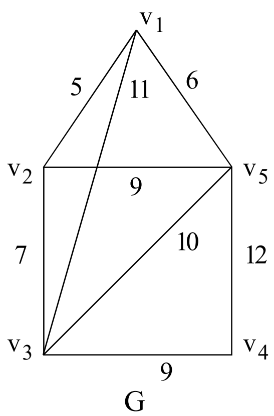
(iii) Find an Euler circuit, if it exists, in the following graph.



2+5+3

(b) (i) Prove that a graph is connected if and only if it contains a spanning tree.

(ii) Find a minimal spanning tree of the following connected weighted graph G by applying Kruskal's algorithm.



G

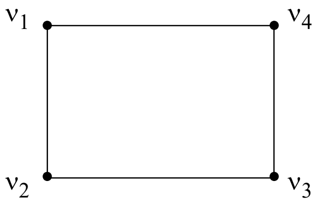
3+7

(c) (i) What is a planar graph?

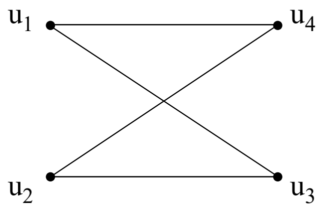
(ii) Let G be a connected planar graph with V vertices, E edges and R regions. Then show that $V - E + R = 2$.

(iii) Show that $K_{3,3}$ is not a planar graph. 1+5+4

(d) (i) Show that the following two graphs G_1 and G_2 are isomorphic.



G_1



G_2

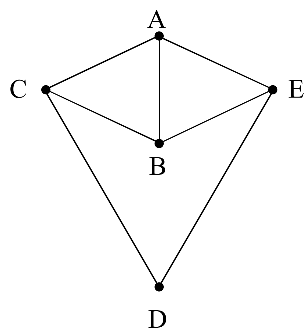
(ii) If A_1 and A_2 are adjacency matrices of G_1 and G_2 respectively, then show that there exists a permutation matrix P so that

$P A_1 P^t = A_2$

where P^t is the transpose of P. 3+7

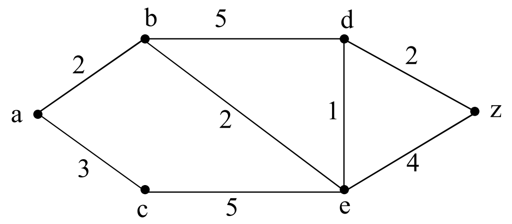
(e) (i) What is Hamiltonian cycle and Hamiltonian graph?

(ii) Is the following graph Hamiltonian? Justify.

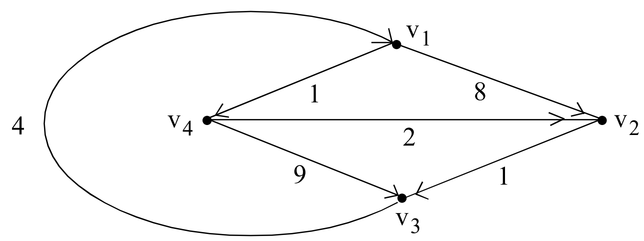


(iii) If a graph G has $n \geqslant 3$ vertices and every vertex has degree at least $\frac{n}{2}$, then show that G is Hamiltonian. 2+3+5

(f) Apply Dijkstra's algorithm to find the length and shortest path between a and z in the following weighted graph. 10



(g) Consider the following directed weighted graph.



Use Floyed–Warshall algorithm to find the shortest path distance between every pair of vertices. 10